An LFR Based on a Ćuk converter for application in HBLEDs

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Abstract

In this paper proposes the design and implementation of a stage voltage converter for use in feed a group of high brightness LEDs (HBLEDs). The use of this lighting technology has been developed over recent years. Currently, these LEDs have high competitiveness in the market despite its high cost, due to its high energy efficiency. This objective must be performed by the voltage conversion stage thus, has been implemented an adaptation stage voltage and current of high performance, in order to introduce the least amount of harmonics on the network. For this use an active power factor correction, based on the Ćuk converter. Using a sliding mode control will be imposed that the converter behaves like a loss free resistor (LFR). Thus, the LFR act as an ideal rectifier power factor near unity.

I. INTRODUCTION

Light emitting diodes (LEDs) are gradually becoming a common source of light [1], [2]. These devices, solid-state lighting is mainly characterized by their long life and high performance. An additional advantage is presented on the CFL, is that the LEDs do not contain contaminants such as mercury [3], and other notable features such as small size, robustness, ease of operation, insensitivity to vibrations and temperature changes, faster ignition, etc...

Taking into account the high performance of these HBLEDs is raising the implementation of a stage adaptation that enhances the performance, and power factor (PF). Some shapes currently studied can be seen in [4], [5]. In this development stage adaptation is studies working as active power factor correction (PFC) in order to minimize distortion in the input current, and make it is in phase with the input voltage. The shape of PFC control is performed using a sliding mode control (SMC) in order to impose on the circuit behaviour as a LFR, [6]. An LFR (Fig. 1) is a two-port network belongs to the class circuits characterized by a balance between the power output and the power input (POPI), which at steady state, behaves as a resistive load. Thanks to this property and their ability to step down the input voltage, the Buck-Boost, SEPIC, modified SEPIC [7] and Ćuk power electronics converters are used as active PFC circuits [8], [9]. In this paper, a Ćuk converter for HBLEDs applications is proposed due to its performance as a AC-DC converter [10].

II. SYSTEM DESCRIPTION AND OPERATION PRINCIPLE UNDER SMC.

Figure 2 shows the general scheme of an ideal single stage AC-DC PFC circuit. The block PFC in our study will be a DC-DC Ćuk converter (Fig. 3).

The control of the system is performed by using SMC [11]. In this case, a sliding surface is chosen in such a way that the input current \(i_r\) is proportional to the input voltage \(v_i\). Therefore, the sliding surface can be described by the following equation:

\[
s(x) := (v_i - R_i) = 0 \tag{1}
\]

It can be observed that if the equation \(s(x)=0\) is imposed by the controller, the input current \(i_r\) will be proportional to the input voltage \(v_i\), leading to a unity power factor, in this way one obtains a unity power factor, so that the grid will “see” the whole system a resistive load “R”. Fig. 2. Furthermore the application of SMC has the effect, in this case, directly linking the state variables: \(i_r\) current witch the input voltage \(v_i\). This
results in a reduction in the order of the system, so having a 4th order system; this reduces their order in a one variable.

Figure 3 depicts the schematic circuit diagram of the Ćuk converter under SMC imposing the sliding condition described by Eq. (1). This scheme shows how the input current \( i \) is the same \( i_1 \) flowing through the inductor \( L_1 \), also has to take into count that the output variables have been inverted with respect to the input, this will be reflected in the graphs of the experimental study.

### III. Mathematical Description

Whereas that the system is working in continuous conduction mode (CCM). Therefore, during each period, the system switches between two different configurations that can be described by a linear system of differential equations corresponding to ON \((u(t)=1)\) or OFF \((u(t)=0)\) states of the MOSFET \( S \). The system state equations can be expressed as follows:

\[
\dot{x} = A_1 x + B_1 u \quad \text{for } u = 1 \tag{2}
\]

\[
\dot{x} = A_2 x + B_2 u \quad \text{for } u = 0 \tag{3}
\]

where \( x = (i_1, i_2, v, v_1)^T \) is the vector of state variables and the over dot stands for derivation with respect to time. Taking into account a model of LED: \( i_e = (v_2 - V_p) / rd \) (where \( V_p \) is the forward voltage and \( rd \) the dynamic resistance), the \( A \) matrices and the \( B \) vectors for the Ćuk converter are given by:

\[
A_1 = \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 1/L_1 & -1/L_1 \\
0 & -1/C_i & 0 & 0 \\
0 & 0 & 0 & -1/rdC_j
\end{bmatrix}
\]

\[
B_1 = \begin{bmatrix}
v/L_1 \\
0 \\
0 \\
V_x / rdC_j
\end{bmatrix}
\]

\[
A_2 = \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & -1/L_1 & 0 \\
1/C_i & 0 & 0 & 0 \\
0 & 1/C_j & 0 & -1/rdC_j
\end{bmatrix}
\]

\[
B_2 = \begin{bmatrix}
0 \\
0 \\
0 \\
0
\end{bmatrix}
\]

Equations (5)-(6) can be combined into a single compact form bilinear expression given by:

\[
\dot{x} = A_1 x + B_1 + [(A_1 - A_2)x + (B_1 - B_2)]u \tag{5}
\]

Defining

\[
A = A_1, \quad \delta = B_2, \quad B = A_1 - A_2, \quad \gamma = B_1 - B_2 \tag{6}
\]

The following bilinear description can be obtained

\[
\dot{x} = (Ax + \delta) + (Bx + \gamma)u \tag{7}
\]

A necessary condition for the existence of a SMC in the surface described by \( s(x)=0 \) is given by the transversality condition [12]:

\[
\langle \nabla s, (Bx + \gamma)u \rangle \neq 0 \tag{8}
\]

The ideal sliding dynamics will be characterised by the conditions

\[
s(x) = 0 \tag{9}
\]

\[
\dot{s}(x) = \langle \nabla s, (Ax + \delta) + (Bx + \gamma)u \rangle = 0 \tag{10}
\]

The second condition (10) defines the equivalent control as the continuous function that constraints the system trajectory to \( s(x) \). It can be also expressed as \( ds/dt=0 \). This condition implies that an expression of the equivalent control can be obtained from the following expression:

\[
\nabla s (Ax + \delta) + (Bx + \gamma)u = 0 \tag{11}
\]

Substituting the \( A \), \( B \), \( \delta \), and \( \gamma \) for their expressions (4) \( y \) (6), and according to (9), the equivalent control for the sliding mode controlled will be given by the following expression:

\[
u_{eq} = - \frac{\nabla s}{V_x} \cdot (Ax + \delta) = 1 - \frac{v_x}{v_i} \tag{12}
\]

By substituting in (7) the discontinuous control \( u \) by the expression of \( u_{eq} \) (12), the equations for the ideal sliding dynamics are obtained. From these equations the following expression for the equilibrium point is obtained:

\[
x_{eq} = \left[ \frac{v_x}{R}, \frac{v_x - V_p}{rd} \right] \tag{13}
\]

The ideal sliding dynamic model is linearized in the vicinity of the equilibrium point assuming slow variations of the input voltage (50 Hz), then a 3rd order characteristic equation for small signal model is obtained:

\[
\Delta(s) = a_1 s^3 + a_2 s^2 + a_3 s + a_4, \quad \text{where:}
\]

\[
a_1 = 1
\]

\[
a_2 = C_1 \left[ R + 2R \sqrt{R + Ro} + Ro \right] + C_1 \sqrt{R \sqrt{R + Ro} + Ro} \tag{14}
\]

\[
a_3 = \frac{L_2 R + \sqrt{R \sqrt{R + Ro} + Ro} \left[ C_1 + C_1 + L_2 + Ro \left( 2C_1 + C_1 \right) \right] \sqrt{Ro \sqrt{R + Ro} + Ro}}{\left( R \sqrt{R + Ro} + Ro \right)^2} \tag{15}
\]

and by applying the Routh-Hurwitz criteria can be determined that the system complies the necessary conditions \( a_i, \delta \) of the same sign -positive-, and sufficiency (14) for the stability.

\[
b_i = \frac{a_2 a_4 - a_1 a_3}{a_3} > 0 \tag{16}
\]

Working with (14) is obtained a sufficient condition as follows \( a_i, \delta > a_i \), in this way can be said that the equilibrium and stability is ensured for the entire input range, since this form has no restrictions. It should be noted that this analysis the input voltage is assumed constant, however the input is time varying and therefore the balance and stability is ensured for the entire input range.

### IV. Simulation Results and Experimental Verification

In order to verify our theoretical results concerning the stability of the system under SMC, time domain numerical simulations have been carried out. An experimental prototype has been also implemented to
validate the numerical simulations. A picture of this prototype is depicted in Fig. 4. The design specifications are given in Table I.

**Fig. 4** Picture of experimental prototype of Ćuk with sliding mode control.

As shown in this table I, an input voltage of 110 V RMS is used. Since that the experimental implementation is constrained by laboratory technical capabilities but the results are also valid for real parameters trough dynamic scaling. This prototype is used to verify the operation and performance of the PFC stage in the study.

To create the equivalent control equation, an hysteretic control with an analog multiplier block to generate the control surface \( s(s)=0 \) is proposed. The switching frequency will vary according to the control law imposed by (1).

![Hysteresis control scheme.](image)

The numerical simulations have been carried out using PSIM package [13]. The parameter values used both in numerical simulations and experimental measurements are shown in Table II.

**Fig. 6** Input signals: \( i_1, v_1 \) and \( P_i \) in steady state operation.

**Fig. 7** shows the steady state behavior of the system obtained from both numerical simulations and experimental measurements. Note that the input current tracks perfectly the input voltage imposing that the circuit "seen" by the source, behaves like a pure resistive load. Thus, it was confirmed that the power factor is very close to unity.

In order to check the quality of the input current, a frequency domain analysis has been carried out. Figure 8 shows the frequency domain spectrum showing the quality of the input current due to the resistive behavior imposed by the SMC. Note that the main harmonic component is at 50 Hz. However, small harmonic distortions at 150 and 250 Hz can also be observed but they can be neglected. In fact, the value of total harmonic distortion (THD) is found to be less 2%.

In a Fig. 8-a it can be observed that the power consumed by the converter and delivered to the load (Fig. 8-b) are very similar which confirms the high efficiency of the system.

### Table I. Design Specifications

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input Voltage</td>
<td>110 V RMS</td>
</tr>
<tr>
<td>Output Current</td>
<td>200 – 700 mA</td>
</tr>
<tr>
<td>Switch Frequency</td>
<td>100 – 300 kHz</td>
</tr>
<tr>
<td>Typical Efficiency</td>
<td>&gt; 85%</td>
</tr>
<tr>
<td>Typical power factor</td>
<td>&gt; 95%</td>
</tr>
<tr>
<td>Led String</td>
<td>3 HBLEDs White in series</td>
</tr>
</tbody>
</table>

### Table II. Main Component Parameters Values of the Ćuk converter for HBLEDs applications.

<table>
<thead>
<tr>
<th>Ref.</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>LED</td>
<td>9 x &quot;Xlamp MC-E&quot; (Cool White)</td>
<td>HBLED: ( V_{cc} = 11.2 ) V ( rd = 2 ) Ω</td>
</tr>
<tr>
<td>L₁</td>
<td>2 mH</td>
<td>Inductor</td>
</tr>
<tr>
<td>L₂</td>
<td>330 µH</td>
<td>Inductor</td>
</tr>
<tr>
<td>C₁</td>
<td>3 x 68 µF</td>
<td>600 V electrolytic capacitor</td>
</tr>
<tr>
<td>C₂</td>
<td>2 x 220 µF + 1 x 47 µF + 1 x 22 µF</td>
<td>100 V electrolytic capacitor</td>
</tr>
<tr>
<td>S₁</td>
<td>FCP11N60F</td>
<td>N-MOSFET</td>
</tr>
<tr>
<td>D₁</td>
<td>USB260</td>
<td>Schottky Diode</td>
</tr>
<tr>
<td>Sliding Control</td>
<td>AD633ANZ</td>
<td>Analog multiplier</td>
</tr>
<tr>
<td>Control</td>
<td>LM311P</td>
<td>Comparator</td>
</tr>
</tbody>
</table>

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can be noted that by varying the load current, the input the output variables under step changes in the load current. It remain unchanged ensuring the LFR operation of the system. and also, the steady state output power and the input power the HBLEDs. This analysis was carried out according to the scheme of Fig. 11, which can verify the feedback of the controller constants are as follows:

\[ P \text{ Controller} \]

The PI controller is described by Eq. (15), in which case the switching variables in SMC (input current, output current and output power) obtained from experimental measurements. The switching frequency is adjusted around 100 kHz for the mean value of the input voltage, (between 70 kHz and 120 kHz), thought the sliding surface \((s(x)=0)\).

The operation of the circuit under load disturbances is also verified. Figure 11 shows the waveforms of the input and the output variables under step changes in the load current. It can be noted that by varying the load current, the input current is unchanged and it is still proportional to the input voltage since this is imposed by the sliding surface \((v=R \cdot i)\), and also, the steady state output power and the input power remain unchanged ensuring the LFR operation of the system.

In addition it has been shown through numerical simulator, the feedback system to control the current flowing through the load, so as to be able to control the brightness of the HBLEDs. This analysis was carried out according to the scheme of Fig. 11, which can verify the feedback of the output current through a control block PI actuating on the variable “R” of the equation of the control surface Eq. (1). The PI controller is described by Eq. (15), in which case the controller constants are as follows: \(k = 1, \tau = RC \equiv 0.5 \mu s\).

\[ G(s) = k \left(1 + \frac{1}{s \tau} \right) \quad (15) \]
Fig. 12 Input signals ($i_i$, $v_i$) and $y$ output current ($i_o$) with the action of the PI controller.

V. CONCLUSION

Design of high-performance power-factor correctors based on Ćuk converters operating in the continuous conduction mode is presented in this paper. A Ćuk converter under sliding mode control and working as an LFR is proposed as an AC-DC adaptor for HBLEDs applications. The proportionality of the inputs signal are accomplished using nonlinear control technique (Sliding mode), this results in a very simple controller design and low input current distortion.

The stability of the system under sliding-mode controller can be guaranteed for the whole range of the input voltage. Numerical simulations and experimental measurements confirm the appropriateness of the proposed system and the sliding mode control for HBLEDs applications.

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REFERENCES