Measuring the thermal conductivity of substrates using the 3 omega method

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Résumé

This work describes the 3 omega method and its use to measure the thermal conductivity of different material. In a first attempt, Cahill’s formula is presented and the linear regime frequency limits are calculated. Numerical simulations using finite element method are done. The results are then compared to Cahill’s analytical solution. It is found that the finite element method solution deviates from Cahill’s at low frequencies. Experimental measurements for Borosilicate substrate are performed and its thermal conductivity is determined.

1. Introduction

The past years have shown an increase in the use of material, such as polymers, (polydimethylsiloxane (PDMS), Kapton etc.) as electronic substrates and in thin and flexible device technologies. It is important to know the thermal conductivity of such material to ascertain its utility for specific applications. The thermal conductivity is defined as the ability of a material to conduct heat. Heat conduction is given by:

\[
\frac{\Delta Q}{\Delta t} = kA \frac{\Delta T}{x} \tag{1}
\]

where \(\Delta Q/\Delta t\) is the rate of heat flow in watts (W), \(k\) is the thermal conductivity in W/m.K, \(A\) is the cross sectional area of the conducting surface in m², \(\Delta T\) is the temperature difference in K and \(x\) is the thickness of conducting surface separating the two temperatures in meters (m).

The 3 omega method is originally developed by Cahill [1] in 1990 where he measured the thermal conductivity of solids. He found out a solution of integral form to show the change of temperature oscillations with respect to angular frequency \(\omega\) over a metallic line heater of very thin width deposited on a substrate. This solution is commonly used to measure thermal properties of material. The 3 omega method has several advantages over other methods. First, it can accurately measure the thermal conductivity. Second, it reduces the equilibration time to few minutes. Finally, the effect of black body radiation is reduced due to the small surface area of the metallic lines [1][2].

The 3 omega method experimental setup is the preliminary setup built in our laboratory. Our setup was first tested through measurements performed on Borosilicate substrate. Numerical simulations using finite element analysis simulation software were done and compared to our measurements. After validating our method, further measurements will be performed on flexible substrates to find its thermal conductivity.

2. Theory

In the 3 omega method, an AC current \(I(t) = I_0 \cos(\omega t)\) at angular frequency \(\omega\) is passed through a metallic line resistance. The metallic line acts as both a resistive heater and a thermometer. Due to joule’s effect, heat will be generated in the metallic line producing temperature oscillations at angular frequency \(2\omega\). Consequently fluctuations in the resistance of the metallic line are produced [3].

\[
R_{\text{metallic line}} = R_0 (1 + \beta_h (\Delta T)) \tag{2}
\]

where \(\beta_h\) is the temperature coefficient of resistance in °C\(^{-1}\), \(R_0\) is the resistance of the metallic line at \(T_0\), and \(R_{\text{metallic line}}\) is its resistance at \(T_0 + \Delta T\). Multiplying the small resistance fluctuations with the alternating current \(I(t)\), ends up with a voltage at frequency \(3\omega\).

![Metallic line with two contact pads deposited on a substrate](image)

Figure 1. Metallic line with two contact pads deposited on a substrate

Figure 1 shows the metallic line deposited on the substrate. The metallic line has two contact pads through which the AC current passes by the aide of micromanipulators.

According to Cahill, the temperature oscillation \(\Delta T\) measured across the metallic line is [1]:

\[
\Delta T_{\text{AC}}(2\omega) = \frac{P_{\text{int}}}{\pi k} \int_0^\infty \frac{\sin^2(\eta b)}{\sqrt{\eta^2 + q^2}} d\eta \tag{3}
\]
where \( p_{\text{rms}} \), \( k \) and \( b \) are the input power per meter of length in W/m, thermal conductivity of the substrate under study in W/m.K, and the half width of the metallic line respectively. In equation (3), \( q \) represents the complex wavenumber of the thermal wave in radians per meter and is given by:

\[
q = \sqrt{\frac{2\alpha}{\pi^2 \omega}} \tag{4}
\]

where \( \alpha \) is the thermal diffusivity in \( \text{m}^2/\text{s} \) of the substrate and is given by:

\[
\alpha = \frac{k}{\rho c_p} \tag{5}
\]

\( \rho \) and \( c_p \) are the density in Kg/m3 and specific heat capacity in J/Kg.K respectively.

Equation (3) stands on several assumptions: The substrate’s thickness \( t_s \) is semi-infinite, the metallic line length is infinite, and the thermal conductivity is isotropic all over the specimen.

The thermal penetration depth is a measure of how deep the thermal waves penetrate into the substrate. It is defined as [4]:

\[
\lambda = \frac{1}{|q|} = \frac{\alpha}{2\omega} \tag{6}
\]

For a thermal penetration depth larger than five times the half width of the metallic line (\( \lambda > 5b \)) and smaller than one fifth of the thickness \( t_s \) (\( \lambda < t_s/5 \)), Cahill has determined a linear behavior for the real part of temperature oscillations with respect to frequency. At this range of frequencies we can define a linear regime. Hence the upper and lower frequency limits of the linear regime can be obtained as [2]:

\[
t_s/5 > \lambda > 5b \quad \rightarrow \quad \frac{25\alpha^2}{4\pi^2} \leq f \leq \frac{\alpha}{100zb^2} \tag{7}
\]

Following the conditions mentioned above, the integral solution, equation (3), can be approximated as [1]:

\[
\Delta T_{AC}(2\omega) = \frac{-p_{\text{rms}}}{2nk} \left( \ln \frac{b^2}{\alpha} + \ln(2\omega) - 1.844 \right) - i \frac{p_{\text{rms}}}{4k} \tag{8}
\]

The temperature oscillation component in-phase with current (real part) decays logarithmically with respect to \( 2\omega \). While the component that is \( \pi/2 \) out of phase with current (imaginary part) is constant over the same range of frequencies. In figure 2 the in-phase and out of phase temperature oscillations are plotted for \( \alpha = 1 \text{ mm}^2/\text{s}, \quad 2b = 20 \text{ µm}, \quad t_s = 1 \text{ mm}, \quad k = 1 \text{ W/m.K}, \quad \) and \( p_{\text{rms}} = 10 \text{ W/m} \). The frequency upper and lower limits of the linear regime are calculated according to equation (7) and found to be:

\[ 2 \text{ Hz} < \omega < 32 \text{ Hz} \]

The third harmonic voltage \( V_{3\omega} \) is related to the temperature oscillation \( \Delta T_{AC} \) as [5]:

\[
V_{3\omega} = \frac{1}{2} V_0 \beta_{\omega} \Delta T_{AC} \tag{9}
\]

where \( V_0 \) and \( \beta_{\omega} \) are the voltage and the temperature coefficient of resistance of metallic line respectively.

Plotting the in-phase temperature oscillation versus \( 2\omega \), the slope in the linear regime can be calculated. Then, the thermal conductivity of the substrate can be computed as follows [6]:

\[
k = \frac{V_0^3 \beta_{\omega}}{4\pi k R_0} \ln \left( \frac{f_2}{f_1} \right) \quad \tag{10}
\]

where \( V_{3\omega,1} \) and \( V_{3\omega,2} \) are the third harmonic voltages for frequencies \( f_1 \) and \( f_2 \) respectively and \( l \) is the length of the metallic line.

3. Simulation

In the previous section, we have used analytical equations to determine the evolution of the temperature oscillation amplitude with respect to frequency over the metallic line. Nevertheless, these equations stand over a number of assumptions such as a substrate with a semi-infinite thickness, a metallic line with a negligible thickness, and an insignificant coefficient of thermal exchange with air.

In order to study the effect of these assumptions, we have developed a numerical simulation based over a finite element method in time domain using COMSOL Multiphysics software. It is a pure numerical method which permits the evaluation of temperature rise on every point in space. This is done through creating mesh on the structure to be studied. When using a metallic line with a length greater than the thermal penetration depth, heat transfer will be approximated as two-dimensional (2D) in a plane perpendicular to the axis of the metallic line. In addition, using symmetry properties makes calculation time shorter.

Figure 3. A sketch (not to scale) of a 10 µm width metallic line deposited over a 700 µm thick borosilicate substrate.
As presented in figure 3, our structure consists of a metallic line of width $2b=10 \mu m$ deposited over a borosilicate substrate of thickness $t_s=700 \mu m$ having a thermal conductivity $k=1.14$ W/m.K, density $\rho=2230$ Kg/m$^3$ and specific heat capacity $c_p=750$ J/Kg.K. The input power applied to the metallic line is 1.5 W/m. The frequency range for the linear regime is between 2.76 Hz and 86.58 Hz. The substrate is placed over a copper plate similar to that used in our measurements. On side faces and bottom of our structure, the temperature is fixed to ambient temperature. Heat is conducted inside the material; however it is convective on the upper surface of the substrate. Therefore, heat exchange with the surrounding is presented by a coefficient of thermal convection ($H=3$ W.m$^{-2}$.°C$^{-1}$). It corresponds to a natural convection over a horizontal surface for a low temperature difference between the heated medium and ambient temperature. Simulation has to be performed for every frequency. Reaching permanent regime (the regime where the temperature oscillation variation is stable with respect to frequency $2\omega$), the amplitude and phase with respect to excitation current of temperature oscillation $\Delta T_{AC}$ are determined.

For maximal temperature oscillation $\Delta T_{AC}(t)$ over the metallic line, the thermal pattern calculated inside the borosilicate substrate at a frequency of 5 Hz is as shown in figure 4.

Figure 5 represents the evolution of the temperature oscillation $\Delta T_{AC}$ (real and imaginary parts) with respect to frequency. We can observe a strong deviation at very low frequencies between the analytical solution proposed by Cahill and the finite element model. This is due to the fact that the substrate has a finite thickness. For other ranges of frequency we can see that the two curves overlap. This confirms the existence of the linear regime from which we can determine the thermal conductivity. Also, we have observed that the parameter $H$ will have a very small influence on the curves as long as its value is low.

Similarly, simulations were done for a metallic line of a 5 µm width over a Kapton substrate of 125 µm thickness. The substrate’s thermal conductivity, density and specific heat capacity are $k=0.12$ W/m.K, $\rho=1090$ Kg/m$^3$, and $c_p=1420$ J/Kg.K. The input power is 1.5 W/m. The upper and lower frequency limits for the linear regime are calculated as: $9.8$ Hz $< f < 39.5$ Hz.

For the finite element method simulations carried out on both Borosilicate and Kapton substrates, it seems that equation (7) is somehow restrictive. The linear zone can be a little widened towards the low and high frequencies.

4. Sample preparation

In order to generate measurable variations of resistance with respect to temperature, the metallic lines must be made of materials having a high temperature coefficient of resistance (TCR) [7]. So, gold is the material of choice because of its high TCR.

The gold metallic lines are deposited over the substrate by photolithography. Initially, the substrate’s surface is cleaned in acetone under ultrasonic waves and then in isopropyl alcohol. This initial step is done to assure that the metallic line will adhere properly on the substrate. After, the substrate is covered with photoresist through a spin coating machine. Next, an optical mask is placed over the photoresist and exposed to ultra violet light. For our negative resist, the photoresist is exposed to light which is now insoluble and the exposed part can be removed by using a developer named AZ 326. The metallization step consists of depositing a 400 nm layer of gold over the substrate surface with a 5 nm layer of chromium in between, to provide good adhesion between the metallic line and the substrate. The procedure is terminated with liftoff where the remaining photoresist is removed.

We designed metallic lines with different widths and length varying between 10 µm and 50 µm, and 3
mm and 16 mm respectively. The resistance of the metallic line depends on its dimensions and varies from 10 to 60 Ω.

5. Experimental setup

As demonstrated in equation (9), temperature oscillations are calculated through the third harmonic voltage \( V_{3\omega} \) across the metallic line. Measuring \( V_{3\omega} \) with respect to frequency, we can determine the thermal conductivity \( k \) of the substrate. Figure 7 shows a schematic diagram of the experimental setup.

![Figure 7. Experimental setup](image)

Our experimental setup is made up of a Wheatstone bridge with the metallic line placed in one of its arms. The Wheatstone bridge is built up of resistors and potentiometers of small temperature coefficient of resistance. The reason is not to generate harmonics that might add to \( V_{3\omega} \) produced by the metallic line. The circuit is supplied by a function generator of low total harmonic distortion.

The metallic line is the single element in the setup that produces third harmonic voltage. Therefore, by varying the potentiometer \( R_3 \), the Wheatstone bridge is balanced where the fundamental voltage \( V_3 \) is suppressed without affecting the third harmonic signal \( W_{3\omega} \). The amplitude and phase of \( W_{3\omega} \) are measured by the aid of an SR830 lock in amplifier. \( V_{3\omega} \) across the metallic line is then obtained as follows [8]:

\[
V_{3\omega} = \left( \frac{R_2 + R_{\text{metallic line}}}{R_2} \right) W_{3\omega}
\]  

(11)

![Figure 8. Experimental measurements of the third harmonic voltage \( V_{3\omega,\text{exp}} \) compared with simulation results \( V_{3\omega,\text{theo}} \).](image)

Experimental measurements were done on a Borosilicate substrate having the same properties as stated in section 3. Figure 8 shows the in-phase and out of phase third harmonic voltage readings (\( V_{3\omega,\text{exp, in-phase}} \) and \( V_{3\omega,\text{exp, out of phase}} \)) plotted with respect to linear regime frequency zone. From simulations done in section 3, the temperature oscillations \( \Delta T_{AC} \) are given and the corresponding theoretical third harmonic voltages (\( V_{3\omega,\text{theo, in-phase}} \) and \( V_{3\omega,\text{theo, out of phase}} \)) are calculated according to equation (9). We can notice a good agreement between the experimental and theoretical measurements. Determining the slope of the linear regime and substituting all the needed values in equation (10), we found out a 1.08 W/m·K thermal conductivity which is not that far from supplier’s reported value of 1.14 W/m·K [9] with an error of 5.2%.

6. Conclusion

In this work we have presented an AC technique, the 3 omega technique, which measures the thermal conductivity of different material. The third harmonic voltage was measured for several frequencies over the calculated linear regime frequency limits. We have obtained a thermal conductivity value for borosilicate similar to that of the supplier’s.

Simulation was performed using finite element method in time domain and then compared to Cahill’s solution. We have verified that both solutions do not overlap at low frequencies. At these frequencies, the thermal penetration depth is in the same order of the thickness of the substrate.

After we had validated our 3 omega experimental setup for solid substrates, further measurements will be done on flexible substrates to determine its thermal conductivity.

7. References